

Loc Sys (\mathbb{D}^*)

"models of lin sys on the punctured disc"

$$dt + g(t)/G(t)$$

$$g \curvearrowright x \rightarrow gxg^{-1} - g^{-1}dg$$

Note: we do not sheafify.

e.g. $G = G_a$, yet g/G . (true for G unipotent)

$$dt + g_a(t)/\mathbb{C}(t)$$

$$g \curvearrowright x \Rightarrow x - dg$$

$$2) G = G_m$$

$$g \curvearrowright A \in \mathbb{R}^\times \times B \in G_m \times \mathbb{C}/\mathbb{Z}$$

$$\text{Hom}(S, G_m(t)) = S(t)^\times$$

$$= a t^i + b(t) t^{i'}, A(t)$$

a invertible $\in S^{\text{nil}}(t)$
 $b \in S \setminus \{0\}$, \mathbb{C} nilpotent.

$$G_m[[t]] = \mathbb{Z} \times G_m \times \prod_{\infty} T/A'$$

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$g \curvearrowright x$: G_m does nothing

\mathbb{Z} : shifts deg

T/A' : changes deg > 0 . component freely w/ ∞ stab.

A_0^{∞} : kills the formal neighborhood of neg part.

$$A_0^{\infty} \times \prod_{\infty} T/A' \times \mathbb{C} / \mathbb{Z} \times G_m \times \prod_{\infty} T/A' \times \hat{A}_0^{\infty}$$

$$= \text{Adp}^{\infty} \times \mathbb{C} / \mathbb{Z} \times \text{B}G_m.$$

3) $G = GL_2$. (or $G_m \times G_a$).

$$dt + \begin{bmatrix} -\frac{n}{t} & t^{n-1} \\ 0 & 0 \end{bmatrix}$$

$$\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \rightsquigarrow \begin{pmatrix} a'(t) - \frac{na(t)}{t} + t^{n-1} b(t) \\ b'(t) \end{pmatrix}$$

Hope: if I know the first terms of my
connection, I know my connection.

e.g. $dt + a(t)$

$$a(t) \in \mathbb{C}[[t]]$$

is always trivial.

(then, $t^{-r} \mathbb{C}[[t]] \rightarrow t^{-r} \mathbb{C}[[t]] / \mathbb{C}[[t]]$

factors through some $t^{-r} \mathbb{C}[[t]] / t^{r+s}$
then they are locally finite type).

This does not hold.

consider the example above.

i.e. $\text{Cosys}(\mathbb{C}^n)$ not (locally) finite type.

$$t^{-r} g[[t]] / \underbrace{(K_{r+s})}_{\text{congruence subgroup}} (t^{r+s} g[[t]]).$$

↓ fibers are Artin stacks.
for large S . (Thm, Sam).

$$t^S g[[t]] / t^{-r} g[[t]].$$

Slopes.

Again, goal: classify $\overset{GL_r}{\text{connections}}$ on D^+ .

Fix connection.

$$\lim_{n \rightarrow \infty} \frac{\dim \left((t\sqrt{t})^n \left(\mathbb{C}[[t]]^{\oplus r} \right) / \mathbb{C}[[t]]^{\oplus r} \right)}{n^r}$$

Lemma. This is gauge invariant.

\Leftarrow Can take any lattice W in the stalk instead of $\mathbb{C}[[t]]^{\oplus r}$.

Lemma RS \Leftrightarrow slope 0.

$dt + t^{-n}$ has slope $n-1$.

Thm.

slope exists and is a rational # w/ denom. $\leq r$.

Pf: (Cyclic vector thm (DeLozue))
 \exists a vector v s.t.
 $v, \nabla v, \dots, \nabla^{r-1} v$
are $\mathcal{O}(t)$ linearly independent.

assume for any v , those are linearly dependent.

those $\nabla^k v$ not in span.

Consider $v' = v + t^k f$ instead.

Corollary every connection is equivalent to the following form

$$dt + \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \\ 0 & \dots & a_s \end{bmatrix}$$

$$\text{slope} = \min \left(0, - \frac{v(a_i)}{r_i} \right)$$

$$= \dots (??)$$

(Something happened).

Can buy into some form -
which induces Sam's theorem.

1. Artinness \longleftrightarrow Fredholmness
of both.

2. Proof that

$k^r[[t]]/G(t)$ is 1-affine.

(3) Consequences for Loc Sys.

Recall $A^1, A^2 \dots$ all 1-affine

but A^∞ isn't

(cotangent complex is infinite)

but does give Loc fully faithful.

(i.e. $T \circ \text{Loc}_{A^\infty} = \text{id}$).

$$\Gamma = \lim T_{A_i}$$

$$\Gamma \circ \text{Loc}(C) = \lim C \otimes_{\text{QCoh}(A^\infty)} \text{QCoh}(A^i)$$

dim across
pullback
& embedding

colimit, good.

↓
colimit

Do same for $g(t)/G(0)$

upgrade to $G(k)$

Why $G(k)/G(0)$ ind-proper.

so QCh wgd

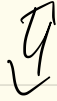
Sam: use $t^{-r}g(t)$ is $G(0)$ -time.

i.e. $C = QCh(\uparrow)$.

$C_{G(0),w} \rightarrow C_{G(0),w}$
is equivalence.

implies $t^{-r}g(t)/G(0)$ is 1-affine.

Remark: $\mathcal{O}_{\text{Coh}}(X) \xrightarrow{\text{monoidal}} \mathcal{O}_{\text{Coh}}(Y)$



$X \rightarrow Y$ descent.



$\mathcal{O}_{\text{Coh}}(X)$ has a geometric
realization in terms
of $\mathcal{O}_{\text{Coh}}(Y)$.

in this case,

$$\begin{array}{ccc} t^{-r}g[[t]] & & \\ \uparrow \pi^? & \downarrow \pi^* & \\ t^{-r}g[[t]]/G_1(0). & & \end{array}$$