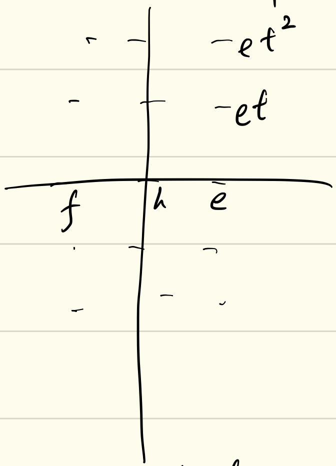
Feb 23. Darrel (Wakimoto Mod)

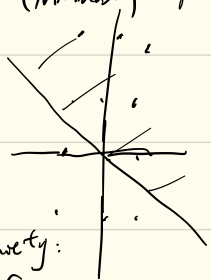
3 flavors of Iwahori.

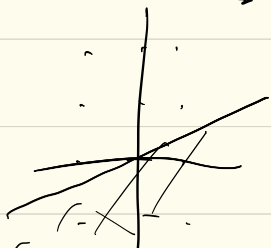


root system of $\widehat{sl_2}$

take $\widehat{sl_2}$ w/ $h, \underbrace{c}_{\text{KM center}}, d = t\partial t$

First kind (Normal) grp | Second ind-grp "Thick" I | Third "semiinf" $N(K)H(O)$





Flag Variety:

$\underbrace{G(K)/I}_{\text{ind-scheme}}$ | $\left(G(K)/I\right)_{\text{scheme}}$ | $G(K)/N(K)H(O)$

Wakimoto

Intuitively: "global sections of D-modules / semi-inf flag variety"

Probably better: GS 的 Factorization modules / $\frac{\infty}{2}$-IC sheaf

c.f. Chiral Principal Series II.

$I^{\circ} = [I, I]$.

I°-orbits in $\frac{\infty}{2}$ flag variety:

Classified by affine Weyl group.

i.e. orbits are $I^{\circ} w N((t)) H\mathbb{C}[[t]]$.

$$w \in W_{aff} = N(H(\mathcal{O}))/H(\mathcal{O}).$$
$$= t^{\lambda} \cdot W_{aff}.$$

Remark:

$K^{0}(D(I \backslash G(K)/I)) \circlearrowleft$ Hecke

= regular rep of Affine Hecke

$$K_0(D(I\backslash G(k)/N(k)H(0)))$$

$=$ periodic modules.

$$G(k)/N(k)T(0) \circlearrowleft \ni t^\lambda$$ translation action.

$$N(I)=I$$
$$N(N(k)T(0))=B(k)$$

let $j_1 := I\cdot 1\cdot N(k)T(0)/N(k)T(0) \hookrightarrow G(k)/N(k)T(0)$. $\hat{g}_k \circlearrowleft$

$\hat{g}_k \circlearrowright W' :=$ " $\Gamma(j_{1*}\mathbb{C})$. "

(negative level, normal I : gives contragredient Verma).

not rigorously defined.

Can fix use $D(G(k))$.

or (as we do) use CADo D_G.

Then the deformation of $j_{!*}\mathbb{C}$ can be made pushforward of $\mathbb{C}_{I^\circ}$.

(More generally, W^u comes from $\mathbb{C}_{I^\circ w I^\circ}$).

and instead we use

$$H^\bullet(\mathfrak{h}[[t]], H^{\frac{\infty}{2}}(\mathfrak{n}((t)), \Gamma(\mathbb{C}_{I^\circ w I^\circ}))).$$

(Morally, "$H^{\frac{\infty}{2}} \circ \Gamma = \Gamma \circ p_*[-\infty]$")

$$p : G(K) \to G(K)/N(K)$$

Compare w/ :

$$\begin{tikzcd} D(G) \arrow[r, "\Gamma_{*,dR}"] \arrow[d, "T^G"'] & D(pt) \arrow[d, "\Gamma"] \\ \mathfrak{g}\text{-mod} \arrow[r, "H_*(\mathfrak{g}, -)"'] & Vect \end{tikzcd}$$

$\left(= \text{IndCoh pushford } pt/\hat{G} \to pt \right)$

Now let's generalize by rewriting as

$$H^{\frac{\infty}{2}}(\mathfrak{n}((t)), \Gamma(-)) \overset{\frac{\infty}{2}}{\underset{\mathfrak{h}[[t]]}{\otimes}} \underbrace{H_0}_{\mathfrak{h}((t))\text{-vacuum}}$$

$\sim$ k-Kost

Generalise: $H^{\frac{\infty}{2}}\left(\mathfrak{n}((t)), T(I^{\circ} w I^{\circ})\right) \overset{\frac{\infty}{2}}{\underset{\mathfrak{n}[[t]]}{\otimes}} R$

$W_k^w(R) \cong$ for R a $\mathfrak{n}((t))_{(-k-k_{crit})}$-mod.

Let's consider crit level:

choice of $R \cong$ choice of $q\mathrm{Coh}$ on $\mathrm{Conn}_{H}(\mathring{D})_{\text{-up-}\rho}$.

want R $\mathfrak{n}[[t]]$ integrable i.e. comes from $\mathrm{Conn}_H(D)$.

will introduce a map

$$\begin{array}{c} \mathrm{Conn}_H(D^{\times}) \\ \downarrow \\ \mathcal{O}_p(D^{\times}) \end{array}$$

(action of FF comes from upstairs).

note: at anti-dominant, non-positive level,

Wakimoto = Verma.

why? we have $G^{\circ} \cong N w_0 B \subset G$.

thus open $B w_0 N / N(\hat{O}) \subset G / N(\hat{O})$

Since our sheaves come from $I_0 w I_0$, $\hookrightarrow G(K)/N(K)$.

Γ suffices to be taken on this open.

There it's easy to describe (as a $\mathfrak{b}$-module).