Tea Structures / ~~.~~ now we talk about char p.

G s.s. $\mathfrak{g}$ Lie alg.
simply connected

Q: BB localization in char p.

$G\text{-rep} \xleftrightarrow{\text{char } 0} U(\mathfrak{g})\text{-rep} \xleftrightarrow{\text{tauto.}} \mathfrak{g}\text{-rep}$.

$h \in \mathfrak{g} \subset \mathrm{Vect}(G) \simeq \mathrm{Der}(\mathcal{O}_G)$.

Consider $h^p \curvearrowright \mathrm{Der}(\mathcal{O}_G)$. $\quad h^p(ab) = \sum_{i=0}^{p} \binom{p}{i} h^i a\, h^{p-i} b = (h^p a) b + a (h^p b)$.

So $\exists\, h^{[p]} \in \mathrm{Der}(\mathcal{O}_G)$ that acts on $\mathcal{O}_G$ in same way as h.

In fact: $h^{[p]} \in \mathrm{Lie}(G)$ moreover, $h^p - h^{[p]} \in Z(U(\mathfrak{g}))$.

Consider $\mathrm{Act}: U(\mathfrak{g}) \to \mathrm{End}(\mathfrak{g})$. $\quad$ By definition, $\mathrm{Act}(h^p - h^{[p]}) = 0$.

$\Longleftrightarrow (h^p - h^{[p]})$ commutes w/ everything.

Consequence: existence of large algebra inside the center.

$\mathrm{Sym}(\mathfrak{g}^{*(1)})$ ← Frob-twist

$\longrightarrow Z(U(\mathfrak{g}))$.

and $U(\mathfrak{g})$ is finite dimensional over $\mathrm{Sym}(\mathfrak{g}^{(1)})$.

On the other hand, $G\text{-rep} \longleftrightarrow U^{\text{divided}}(\mathfrak{g}) = \left\{ \dfrac{h^{p^i} - h^{[p^i]}}{p^i!} \right\}\text{-rep}$ ← aka "the hyperalgebra".

$\updownarrow$

$\mathcal{O}_G\text{-comod}$ ← char p dualization.

Exercise: prove for $G = \mathbb{G}_m$.

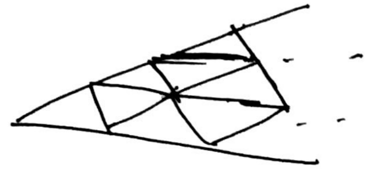
$U(\mathfrak{g}) \xrightarrow{\text{surj}} U^{\text{small}}(\mathfrak{g}) = U\mathfrak{g}/Z\mathfrak{g} \longrightarrow U(\mathfrak{g})^{\text{divided}}$
(image)

Rep theory understanding of $\mathfrak{g}$ ~~...~~ rep of G. $\quad$ Turns out $U\mathfrak{g}^{\text{div}}$-rep is controlled by $U(\mathfrak{g})$-rep.

G-rep is parametrized by dominant weights (but not semisimple!

Linkage principle via Affine Weyl.

(lines are p-root hyperplanes).



For GL_n, $(n-1)!$-blocks suffice to control all of G-rep.

(For sufficiently small block, restriction of irrep gives irrep).

Lusztig's Conjecture:

$$[\Gamma(\mathcal{O}(\mu)) : L(\nu)] = h_{w_0\nu, w_0\mu}(1).$$

affine KL polynomial

for p large enough known by AJS, B, F.

doubly exponential.

$$\text{modular} \xleftrightarrow{\text{char } p} \mathrm{QCoh}(T^*G/B) \xleftrightarrow[\text{equivariant ver.}]{\text{char } 0} \text{affine}.$$

$$\mathcal{D}\text{-mod}(G/B) \simeq U(\mathfrak{g})_0\text{-mod}.$$

derived

$$\boxed{D^b(\mathcal{D}\text{-mod}(G/B)) \simeq D^b(U(\mathfrak{g})_0\text{-mod})}$$ ①

Note: $\mathcal{D}$ = "crystalline D-alg" $= U_{\mathcal{O}_{G/B}}(\mathrm{Vect}(G/B))$.

Cool stuff: Like before, $\mathcal{D}$ has a large center.

② $Z(\mathcal{D}) \simeq \mathcal{O}(T^*)$

$\exists$ Azumaya alg (Frob twist ignored) $\mathcal{A}$ on $T^*(G/B)$.

s.t. when push to G/B, get $\mathcal{D}$-back

Thus $\boxed{\mathcal{D}\text{-mod}(G/B) \simeq \mathcal{A}\text{-mod}(T^*)}$

$\forall G$ Vect, $V^P - V^{[P]} \in Z(D)$.

Proof of ① $U(\mathfrak{g})_0$-mod $\underset{\Gamma}{\overset{loc}{\rightleftarrows}}$ D-mod (G/B).

Loc is ff: $\Gamma \circ Loc = Id$. Pf: ATS: $R\Gamma(D)$ is $U(\mathfrak{g})_0$.

Pf: look at associated graded.

$R\Gamma(D) \rightsquigarrow R\Gamma(\mathcal{O}_{T^*G/B})$. only in deg 0. (Cite Gremert-Schroeder)

now check characters.

$Loc(U(\mathfrak{g})_0\text{-mod}) \subset D(D\text{-mod}(G/B))$.

consider $Loc(\ )^\perp$ = objects having no map from Loc.

$= \{M \mid \Gamma(M) = 0\}$.

$D\text{-mod}(G/B) = \langle loc, Loc^\perp \rangle$.

semiorthogonal decomp.

claim: $\nexists$ nontrivial semiorthogonal decomp satisfying some conditions.

$Hom(A, B) = 0 \Rightarrow Hom(B^\vee, A^\vee) = 0$.

each subcat is closed under duality. So it'd be a direct decomposition.

claim: the category is connected (look at skyscrapers)

② is now almost obvious.

$U(\mathfrak{g})_0\text{-mod} \simeq D\text{-mod}(G/B) \simeq A\text{-mod}(T^*)$

$\circlearrowleft$

$Sym\, \mathfrak{g}^*$ on everything.

For any $x \in N$, A splits on $Sp^{-1}(x)$ in fact on formal neighborhood.

(i.e. is a matrix alg) Springer resolution.

$U(\mathfrak{g})_{0,\hat{x}}\text{-mod} \simeq \cancel{A}\ QCoh_{\hat{x}}(T^*)$.

Now back to Ten structures

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this talk → t-structure on $QCoh(T^*)$ for positive char. fields.

this semester → — — — $\cancel{QCoh^G(T^*G/B)}$. ... char 0 fields. } how to compare?

key:

$\mathrm{Braid}_{\mathrm{aff}} \curvearrowright QCoh(T^*)$ (or its equivariant version).

$\mathrm{B}_{\mathrm{aff}} \longrightarrow D(I^\vee \backslash G^\vee(K)/I^\vee) \longrightarrow D^G(St)$

$\sigma_i \rightsquigarrow j_{i,!}$ (or $*$)

An exotic coherent t-structure on $QCoh(T^*)$ (or equivariant version) is one satisfying $\pi_*: T^* \longrightarrow N$ is exact.

Action of B^+_{aff} is right(?) exact.

B^-_{aff} -- left — — .

~~stacks that ext~~

⇓

Thm 1 $\exists!$? (at most one) exotic t-structure. ($\mathcal{O}$ is proj by assumption produces all projectives now check they yield a t-structure)

Thm 2 $\exists$ such t-structure.

For k positive char. apply ~~to~~ derived BB.
and fixed ~~a~~ nilpotence

This half works for any char $> h^\vee$.

Reduce general to above.

checks of ext vanishing can be done 1) can do ~~[illegible]~~ on formal neighborhood.
2) as finite dim. v.b.

exotic perverse t-structure

works for char 0 and G-eqv. only

Rmk:
π_* sends $\heartsuit$ to perverse coherent sheaves (how to define?
note nilpotent orbits have even dimensions).

(In fact:
(Comes from Affine flag variety))

Grothendieck coherence means that push to $\mathfrak{g}^*(\operatorname{Spec}\mathbb{Z})$ is supported on ~~closed~~ proper subset.

How to relate these two t-structures?
- For a specific strata, perv. t-structure becomes coh.
  (This is used for Lusztig's conj).