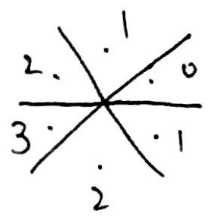
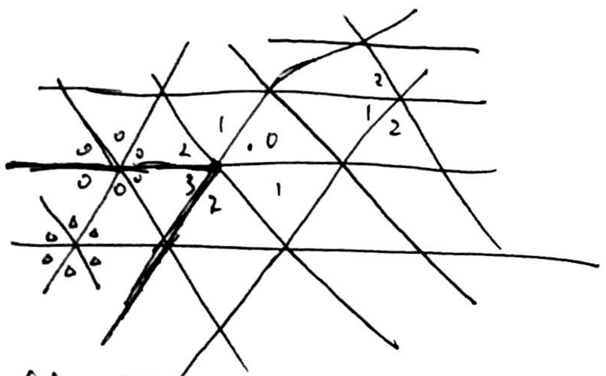


Flag variety of $SL_3 = \bigsqcup_w BwB$.

$SL_3/B = \bigsqcup BwB/B$. (Look at figure on the left).

$\dim BwB/B = \ell(w)$ affine space.

$Fl := PGL_3((t))/I$.



each shape is a connected component.

Goal: (from AB)

Iwahori-Whittaker on $Fl \simeq QCoh^{\check{G}}(\tilde{T}^*\check{G}/\check{B})$.

Recall: $I^\circ < I < G((t))$. $\quad I^\circ/[I^\circ, I^\circ] \supset \check{N}/[\check{N},\check{N}]$ (s.d.). take a generic χ of N $\rightsquigarrow$ a character on I°.

$Iw := D^{I^\circ,\chi}(Fl)$. (Note: AB uses I^- instead).

$F \in Iw$, stalk at origin is trivial: $Iw \xrightarrow{i_0^*} D^{I,\chi}(pt) \simeq 0$.

which $w \in W_{aff}$ can an obj $F \in Iw$ actually have a nonzero stalk at?

those. $|W_{aff}| = |\Lambda| \times |W_{fin}|$.

$|\text{life-supporting planets}| = |W_{aff}|/|W_{fin}| = |\Lambda|$.

We've constructed $\mathcal{O}_\lambda$ for $\lambda \in \Lambda$. in Iw.

Goal: (before)

step 1. functor $QCoh^{\check{G}}(\tilde{T}^*\check{G}/\check{B}) \longrightarrow D^I(Fl)$. — done before

step 2: show it's equivalence.

Step 1: Z_λ : Rep $(\check{G}) \xrightarrow{\text{Satake}} I(Fl)$.

J_λ : Rep $(\check{T}) \xrightarrow{\text{Wakimoto}}$

$j_\lambda := j_{\lambda,*}$ for $\lambda \in \Lambda^+$.

for $\lambda = \lambda_1 - \lambda_2$. $\lambda_1, \lambda_2 \in \Lambda^+$.

$j_\lambda := j_{\lambda_1,*} * j_{\lambda_2,!}$.

Together this supplies sh. Cat $/ pt/\check{G}\times\check{T}$.

want. action of $(T^* G/B \times_{G/B} G/U \,/\, G\times T)$.

$\mathring{E} := \tilde{T}^* G/U$ is the subvariety of $G/U \times \mathfrak{g}$ given by requirement that $b \in \mathrm{Ad}_a \mathfrak{u}^{-}$!
(a, b)

E is the subvariety of $\overline{G/U} \times \mathfrak{g}$ given by considering $G \curvearrowright \overline{G/U}$

E is affine.

so $\mathfrak{g} \xrightarrow{\varphi} \mathrm{VectField}(\overline{G/U})$

take $(a, b \mid \varphi_b|_a = 0)$.

$D(Fl) \otimes_{pt/\check{G}\times\check{T}} pt \simeq \{ F \in D(Fl)$ w/ iso $Z_\lambda * F \simeq V_\lambda \otimes F$, $J_\lambda * F \simeq \mathbb{C}_\lambda \otimes F \}$.

$\circlearrowright$

$E \Leftrightarrow$

$Z_\lambda \xrightarrow{b_\lambda} J_\lambda$ for any λ.

$Z_\lambda \xrightarrow{N_\lambda} Z_\lambda$ nilpotent. s.t. $b_\lambda \circ N_\lambda = 0$.

To show it lives over $\mathring{E}/G$, check some Koszul complex vanishing.

Step 2: equivalence.

$$\Phi: QCoh^{\check{G}}(\tilde{T}^*\check{G}/\check{B}) \longrightarrow D^I(Fl) \xrightarrow{f} D^{I_0,\chi}(Fl).$$

$$\downarrow \wr$$

(AB:) $\xrightarrow{Av} D^{I_0^-,\chi}(Fl)$

f is convolution of $\Delta^0 \in {}_I\backslash G(K)/(I^-,\chi)$. $\Delta^0 = \text{push}({}_I\backslash D(O)/I_0,\chi)$

$\simeq D(B\backslash G/N,\chi) \simeq Vect$.

Claim: Φ is essentially surjective.

Reason: $\Phi(O(\lambda)) \simeq f(J_\lambda)$ is supported on the closure of Fl_w

for $w \in W_{fin} \cdot \lambda$ of highest length.

and it has nontrivial (1-d) stalk on w.

Thus 1-1 correspondence between (coweight) $\longleftrightarrow$ (life-supporting region).

Remains to show: $Hom^\bullet_{T^*\check{B}/\check{G}}(V_\lambda, O(\mu)) \simeq Hom^\bullet_{D^{I^-,\chi}(Fl)}(\Phi(V_\lambda), \Phi(O_\mu))$.

Claim: this implies AB.

Pf: above implies $Hom(V_\lambda \otimes O(-\mu), O)$ is equal on both sides. $V_\lambda \otimes O(-\mu)$ generates $D^{\check{G}}(T^*\check{G}/\check{B})$.

$\Rightarrow \Phi$ is ff on pairs (F, O)

$\Rightarrow \Phi$ is ff on pairs $(F, O(\lambda))$.

b/c $O(\lambda)$ generate $D^{\check{G}}(T^*\check{G}/\check{B})$.

$\Rightarrow \Phi$ is ff.

$\mathrm{Hom}_{T^*\check{B}/\check{G}}(V_\lambda, \mathcal{O}(\mu)) \simeq \mathrm{Hom}^{\check{G}}(V_\lambda, H^\cdot_{T^*\check{G}/\check{B}}(\mathcal{O}(\mu)))$.

Claim: $H^i = 0$ for $i > 0$. Pf: Grauert–Riemann-Schneider.

$\wr\wr$

$H^i(K \otimes \mathcal{O}_\mu) = 0$ (compare: Kodaira vanishing).

So LHS $= \mathrm{Hom}^{\check{G}}(V_\lambda, H^0(\mathcal{O}_\mu)) \simeq \mathrm{Hom}^{\check{G}}(V_\lambda, \oplus H^0(\mathrm{Sym}^\cdot T \otimes \mathcal{O}(\mu)))$ ← tangent bundle

has dim $[\mu : V_\lambda]$. $\quad ||$

Pf:

$\mathrm{Ind}_B^G \mu \otimes \mathrm{Sym}\, n$

~~Hom~~ $[\lambda, \mu \otimes \mathrm{Sym}\, n]_G^{\otimes} \overset{?}{=} [\mu : V_\lambda]$.

"

~~dim Hom (…)~~

~~dim Hom (~~

$\mathrm{Hom}(\Phi(V_\lambda), \Phi(\mathcal{O}_\mu))$. (*)

Claim: $\Phi(V_\lambda)$ is tilting.

Pf: (from AB): 1) * for $V_1, V_2 \to$ * for $V_1 \otimes V_2$.

2) * for minuscule & quasi-minuscule via pain.

3) general fact: everything is a summand in (tensor of minuscule and quasi-)

2nd proof: Koszul: simple $\longleftrightarrow$ tilting.

Whittaker $\longleftrightarrow$ Parabolic.

Then $KD(\mathrm{Whit}(Z_\lambda)) \simeq \pi_{*\, Fl \to Gr}(Z_\lambda) = IC_\lambda$.

Unproven claim: Z_λ is Koszul self-dual

Claim $\Rightarrow \mathrm{Hom}^1$ is trivial.

To compute Hom^0, look at K-theory, use Wakimoto filtration

Now we still need to check $\mathrm{Hom}^\bullet \longrightarrow \mathrm{Hom}^\bullet$ is iso $\Longleftrightarrow$ inj.

take $T^*\check{G}/\check{B} \supset \mathcal{U}$. $\mathcal{U}$ is the place where $\tilde{N} \to N$ is iso.

$\mathrm{Hom}(V_\lambda, \mathcal{O}(\mu))$

$\downarrow$ inj

$\mathrm{Hom}_{\mathcal{U}/G}(V_\lambda, \mathcal{O}(\mu))$.

$\mathcal{U}$ is a single $\check{G}$-orbit. whose stabiliser looks like an abelian subgroup of N.

$\mathrm{Hom}_{D^{I^\circ,\chi}(Fl)}(\Xi, \Xi)$

$\downarrow$ inj

$D^{I^\circ,\chi^+}(Fl)^{\text{thick}}$ $\simeq C^{\text{stupid}}$

$\curvearrowleft$ $\mathcal{I} \subset G[t^{-1}]$

$:= D^{I^\circ,\chi^+}(G(K)/\mathcal{I})$

$\underline{G(K)/\mathcal{I}} \hookrightarrow \mathrm{Bun}_G(\mathbb{P}^1)$ w/ one flag.

$C^{\text{stupid}} = D_{I^\circ,\chi}(Fl) / (\text{all nonthermal simples})$.