

Goal: to access singular support more easily (concretely).

Key observations:

- 1) For C ~~nice~~ ^{sym. monoidal}, $C \in C$ compact generator, we have $C \cong \text{Incl}(C)$ -mod.

$$M \mapsto \text{Hom}(C, M)$$

(Functorial).

$$C \otimes_{\text{Incl}(C)} N \leftarrow N$$

In particular, if $\mathbb{1}$ is compact generator, then

$$C \cong \text{HC}(C)\text{-mod.}$$

(namely, $C = \text{InclCoh}(X)$)

for $X = \text{pt} \times \text{pt}$.

- 2) Support of modules \longleftrightarrow Sing supp of sheaf.

- 3) In nice case, $\text{HC}(C)$ is particularly easy.

Applications:

- 1) ~~Compact~~ ^{Generators} ~~generators~~ ~~Methods~~ of InclCoh_X .

- 2) $\text{QCoh} = \text{InclCoh}_0$.

We observed C , so need converse.

$$\text{HC-mod}_0 \xrightarrow{\sim} \text{InclCoh}_0 \hookrightarrow \text{InclCoh}$$

Reinterpreted the composite functor

then show its ~~compact~~ generators lie in image of \exists

- 3) Geometric Serre. (no time).

General Background.

A. By groupoid we mean simplicial objects $\dots R \times_X R \rightrightarrows R \rightrightarrows X$

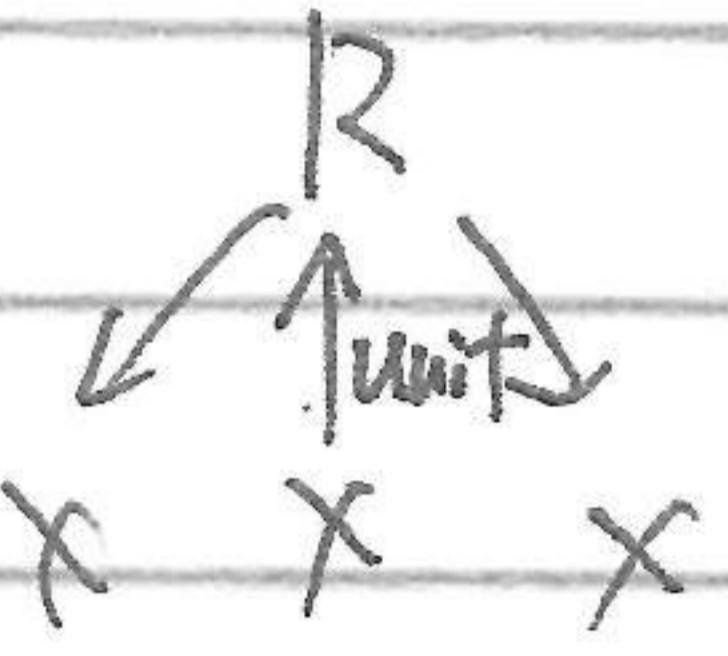
and write $\begin{matrix} & R & \\ \swarrow & & \searrow \\ X & & X \end{matrix}$ for simplicity.

initial obj: X (diagonal groupoid)

final obj: $X \times X$ (free groupoid)

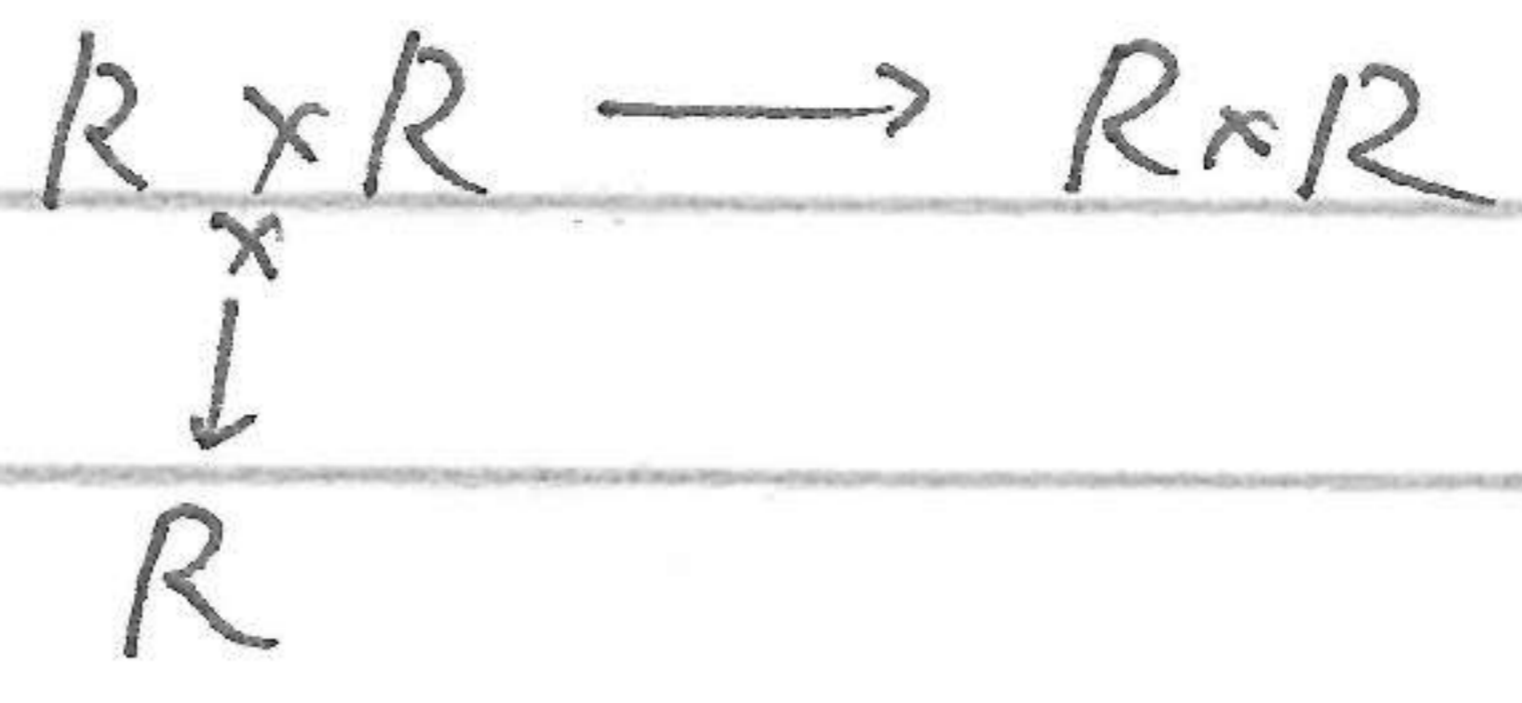
For this talk!

↓
 (really, R, X
 both affine schemes).



In this talk further: $R \xrightarrow{\text{target}} X$ and $R \times_x R \xrightarrow{m} R$ are schematics

Then 1) $\text{InelCoh}(R)$ admits monoidal structure.

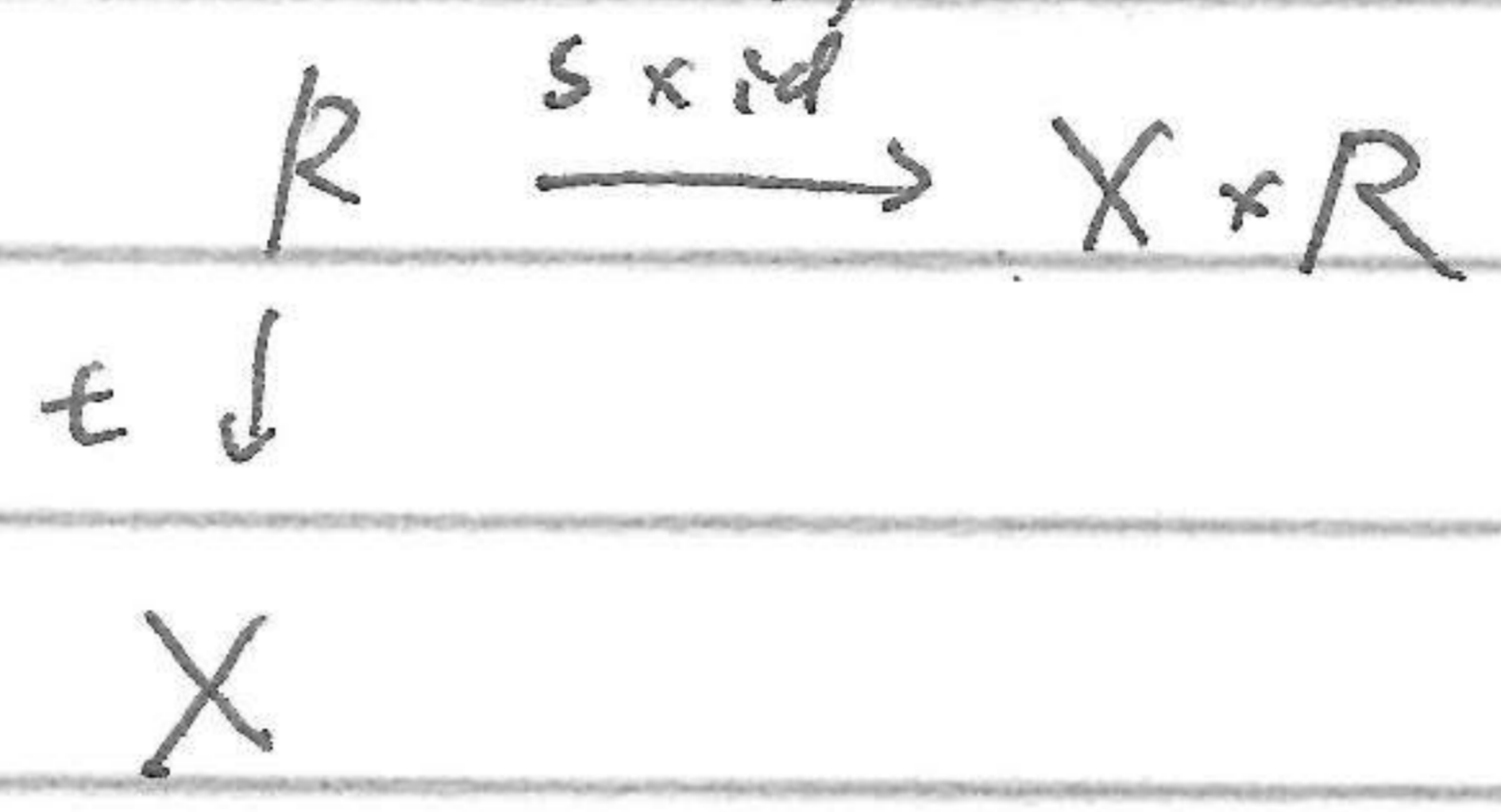


Example: free groupoid $X \circledast X$

$\rightsquigarrow \text{InelCoh}(X \circledast X)$

$\cong \text{Funct}_{\text{cont}}(\text{InelCoh}(X), \text{InelCoh}(X))$

2) $\text{InelCoh}(R) \rightsquigarrow \text{InelCoh}(X)$



Thus $\text{HC}(\text{InelCoh}(R))$ is an E_2 -algebra.
 $= \text{Inel}(\text{unit}_x^{\text{IC}}(w_x))$.

(Remark: use $\mathcal{Q}\text{Coh}$ gives the same E_2 -alg)

B. Recall the general phenomenon explained in WEU seminar:

$$\text{LMod}_C(C) : E_n\text{-alg}(C) \rightarrow E_{n-1}\text{-alg}(C\text{-ModCat})$$

is fully faithful.

More precise ver: \exists fully faithful sym monoidal

$$E_1\text{-alg} \rightarrow \text{pointed DG Cat}_{\text{cont}}$$

$$A \mapsto (A, A^{\text{op-mod}})$$

Right adj:

essential image:

$$(c, C)$$

C compactly generated

$$(c, C) \mapsto \text{End}_C(c, c)$$

right-lax \rightsquigarrow upgrade to map of algebras.

$$E_1\text{-alg}(\text{DG Cat}) \rightarrow E_2\text{-alg}$$

Concret: $\mathbb{O} \rightsquigarrow (\mathbb{1}_0, 0) \rightsquigarrow \text{End}_0(\mathbb{1}_0) \in E_2\text{-alg}$

General nonsense \Rightarrow

$$\text{End}_0(\mathbb{1}_0) \xrightarrow{\text{rat-op}} \text{non-unital}$$

~~chain~~ iso iff $\mathbb{1}_0$ compact generator.

(fully faithful \Rightarrow idempotent)

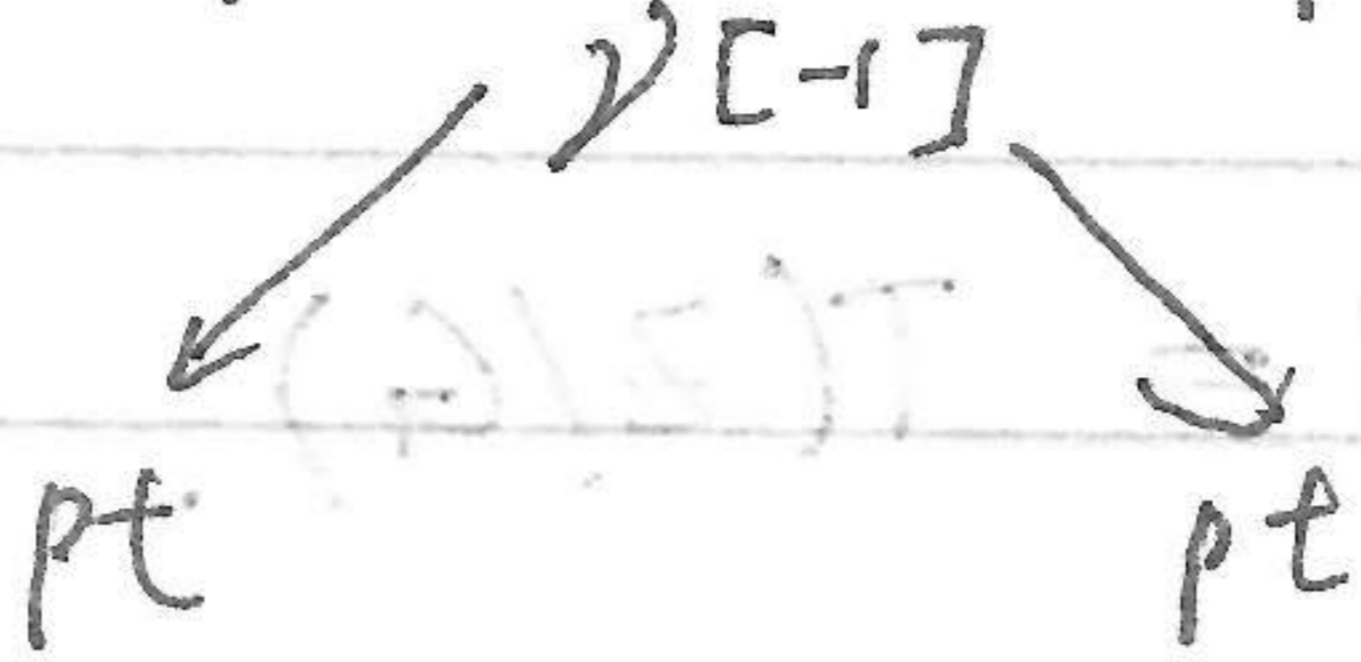


3 (Content on Back)

This is something: a ^{nice} sym. monoidal $\mathcal{C}at$ can be recovered from its center!

What's a good example? $\mathcal{V}[-1]$ for \mathcal{V} affine smooth scheme.

i.e.



this is a group scheme.

[Add: $\mathcal{V}[-1]$].
See Page 13

Any quasi-smooth $X=pt$ looks like this!

Let $\Delta_{pt}: pt \rightarrow pt \times pt$ be the diagonal.
($\mathcal{V}=V: \text{Spec Sym } V^*[1]$).
 $= \mathcal{V}[-1]$. (This is closed embedding.)

The \mathbb{I} here is $(\Delta_{pt})_*^{IC}(K)$. Claim:

- 1) This generates $IC(\mathcal{V}[-1])$ (because underlying classical reduced is a pt - 4.1.8).
- 2) This is compact. (need: $\text{IndCoh}(S^0) = 0$?)

Then story above yields:

$$\text{IndCoh}(\mathcal{V}[-1]) \simeq \text{HC}(\text{pt}/\mathcal{V}) \simeq \text{End}(\Delta_{pt}^* K)$$

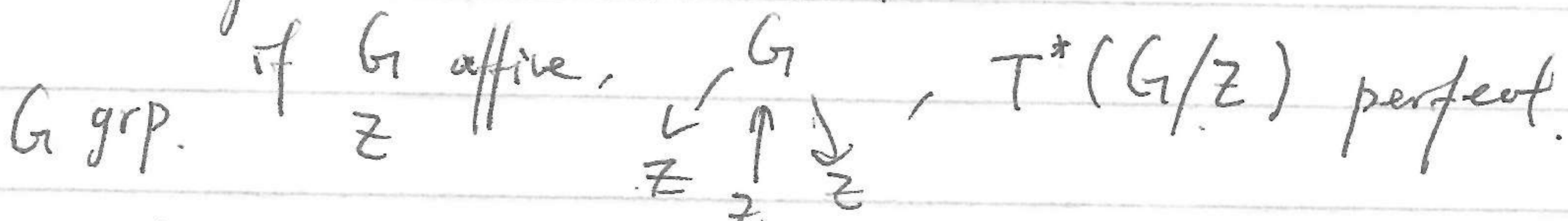
So we're back to our orig. supp. setup. To proceed, we need to compute $H^0 \text{HC}(\text{pt}/\mathcal{V})$. This is given by:

$$\text{HC}(\text{pt}/\mathcal{V}) \simeq \text{Sym}(V[-2]), \quad V = T_{pt} \mathcal{V}.$$

That is to say, $\text{Spec}(HH) = V^*$. (as E_1 -algebras!)

I need to explain why this is the case.

The general statement is that:



Then $\text{HC}(\text{IndCoh}(G)) \simeq \Gamma(Z, \text{Sym}_{O_Z}(L_G))$

(Instantiation: see back)

$$L_G \triangleq \text{unit}^*(T(G/Z)[-1])$$

\uparrow
 $\text{QCoh}(Z)$

(as E_1 !)

$$T_{X/Y} \rightarrow T_{X/Z} \rightarrow f^k T_{Y/Z}$$

3 (Back)

Namely, $\text{unit}^*(T(G/p_t)/[-1])$

$$= T(p_t/G)$$

$$= [K \rightarrow 0 \rightarrow V]$$

$$T_G = V[-1]$$

$$\Phi: \text{unit}^*(T(G/Z)/[-1]) = T(Z/G) \quad (X=Z, Y=G)$$

[Faint handwritten notes and diagrams, including arrows and mathematical symbols, are present in this section.]

This follows from two observations:

- 1) $L_G \in \mathcal{Q}\text{Coh}(Z)$ has a Lie alg structure.
- 1a) This structure is trivial when G is grp.
- 2) $\exists T(Z, \mathcal{U}_{O_Z}(L_G)) \rightarrow HC(\text{InclCoh}(G))$
which is iso when Z is eventually conn.

Use this

General setup:

(G.2.2) $i: Z \rightarrow W$ proper between affine DG scheme. $W = G$
 $s: W \rightarrow Z$ st. $s \circ i = \text{id}_Z$. $i: Z \rightarrow G$
 $T^*(Z/W)$ perfect. $s = p_1$

$\rightsquigarrow T(Z/W) \in \mathcal{Q}\text{Coh}(Z)^{\text{perf}}$

then: 1) $T(Z/W)$ has Lie structure.

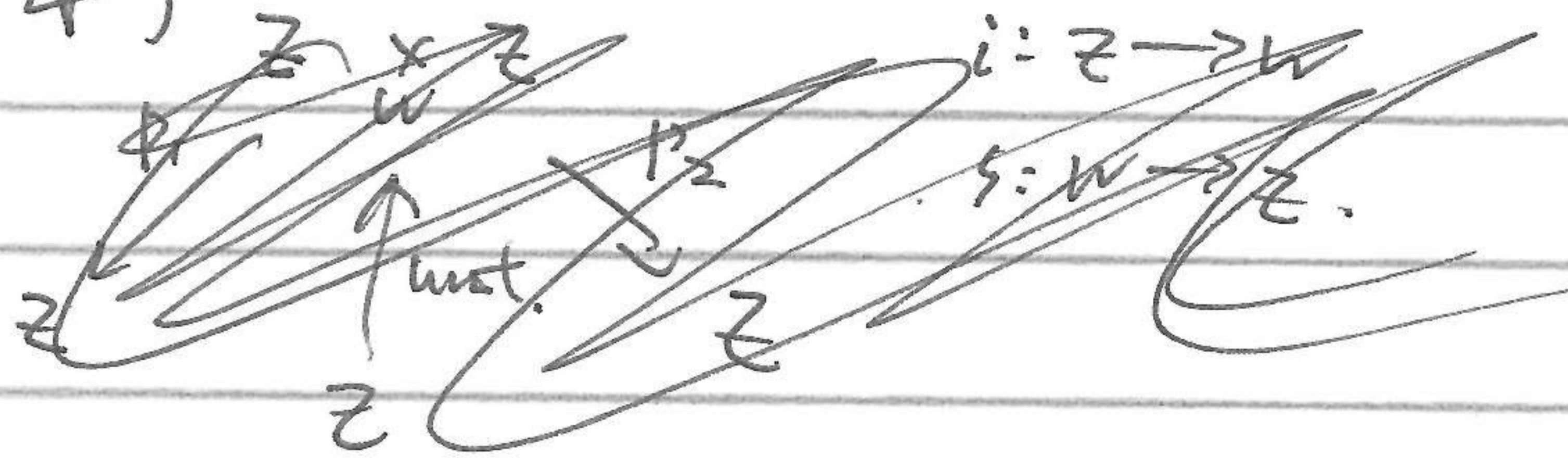
2) $\frac{T(Z, \mathcal{U}_{O_Z}(T(Z/W)))}{\text{alg}} \rightarrow HC(\text{InclCoh}(W))$
eventually connective.

This is because

$L_G \cong \Omega(\text{unit}^*(T(G/Z)))$
 all loops of Lie are trivial.
 $Z \times_Z Z \cong \Omega(G)$ as grp / Z .

G.2.2's proof:

(Start w/ \star)



1) $\text{Hom}(i_* \mathcal{F}, i^* \mathcal{G}) \cong \text{Hom}(i_* \mathcal{F}, p_2^* \mathcal{G})$

$Z \times_Z Z \rightarrow Z$

Base Change

$Z \rightarrow W$

~~Thus $\text{End}(i_* \mathcal{F}) \cong \text{Hom}(i_* \mathcal{F}, p_2^* \mathcal{F})$~~

\star : retraction $\Rightarrow Z \times_Z Z$ is group over Z .

In general: $\text{unit}^*(T^*(G/Z)) \in \text{colie}(\mathcal{Q}\text{Coh}(Z))$.

AFT: eventually connective, w/ coherent cohomology.
 $\in \mathcal{Q}\text{Coh}(Z)^{-}$

grp scheme: in $\mathcal{Q}\text{Coh}^{\text{perf}}$

$$G = Z \times_w Z.$$

u^* of $T^{IC}(G/Z) := D^{Serre} T^*(G/Z) \in \text{Lie}(IC(Z)).$
 (In fact in Perf_Q) u^* of $T(G/Z) := D^{naive} T^*(G/Z) \in \text{Lie}(QCoh(Z)).$ $\leftarrow \otimes_{\mathcal{O}_Z} \omega_Z.$

1a) $u^* T(G/Z) = T(Z/w).$
 follows (usual tangent computation).

2) Base change: $i_1^* \circ i_*^{IC} = p_{2*}^{IC} \circ p_{1!}$ $Z \times Z \rightarrow Z$
 Thus $\text{Hom}_{IC(Z)}(w_Z, p_{2*} p_{1!} w_Z) \stackrel{\text{as Gr-alg}}{=} \text{Incl}(i_* w_Z)$ $Z \rightarrow w$

$p: G \rightarrow Z = U$
 is iso on underlying scheme

$$\text{Hom}_{IC(Z)}(w_Z, p_{2*} w_G^\wedge)$$

$w_Z^\wedge \in \text{InclCoh}(G)_Z$ (set-the. supp)

$$IC(G)_Z \xrightarrow{\quad} IC(G)$$

general: $\text{Hom}_{IC(Z)}(w_Z, U(T^{IC}(G/Z))) \text{Distr}(\exp(L)) \subseteq U(L).$

γ_Z fully faithful when Z eventually connected.

$$\Gamma(Z, U(\text{unr}^*(T(G/Z)[-1]))) \subseteq \text{Hom}_{QC(Z)}(\mathcal{O}_Z, U_Z(T(G/Z))) \xrightarrow{\gamma_Z} \text{Hom}_{IC(Z)}(w_Z, U(T^{IC}(G/Z)))$$

as ~~class~~ alg.

Back to main story. We know now $H^0(V[-1]) = \text{Sym}(V[-2])$

write $KD : IC(V[-1]) \rightarrow HC(pt/y) \text{-mod}$

$\mu \mapsto \text{Hom}(O_{pt} \otimes k, \mu)$

Now we can define ~~support~~ support. (i.e. via $HC(pt/y) \rightarrow IC(V[-1])$)

Claim: ~~support~~ defined in this way is that of the support of the $\text{Sym}(V[-2]) \text{-mod}_{gr} H^i KD(-)$.

Pf: (3.4.4) (compactly general case can be checked on obj.)

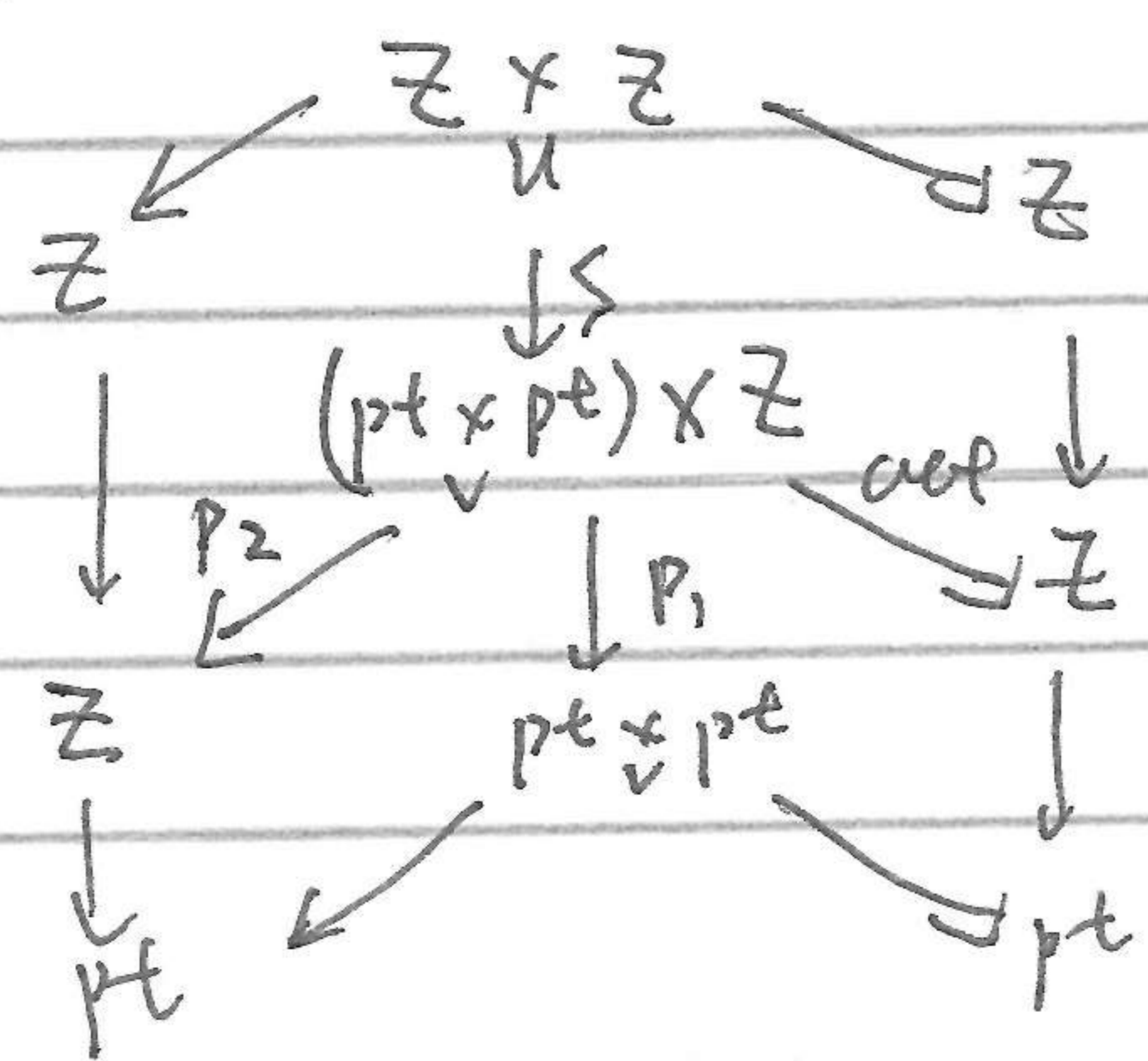
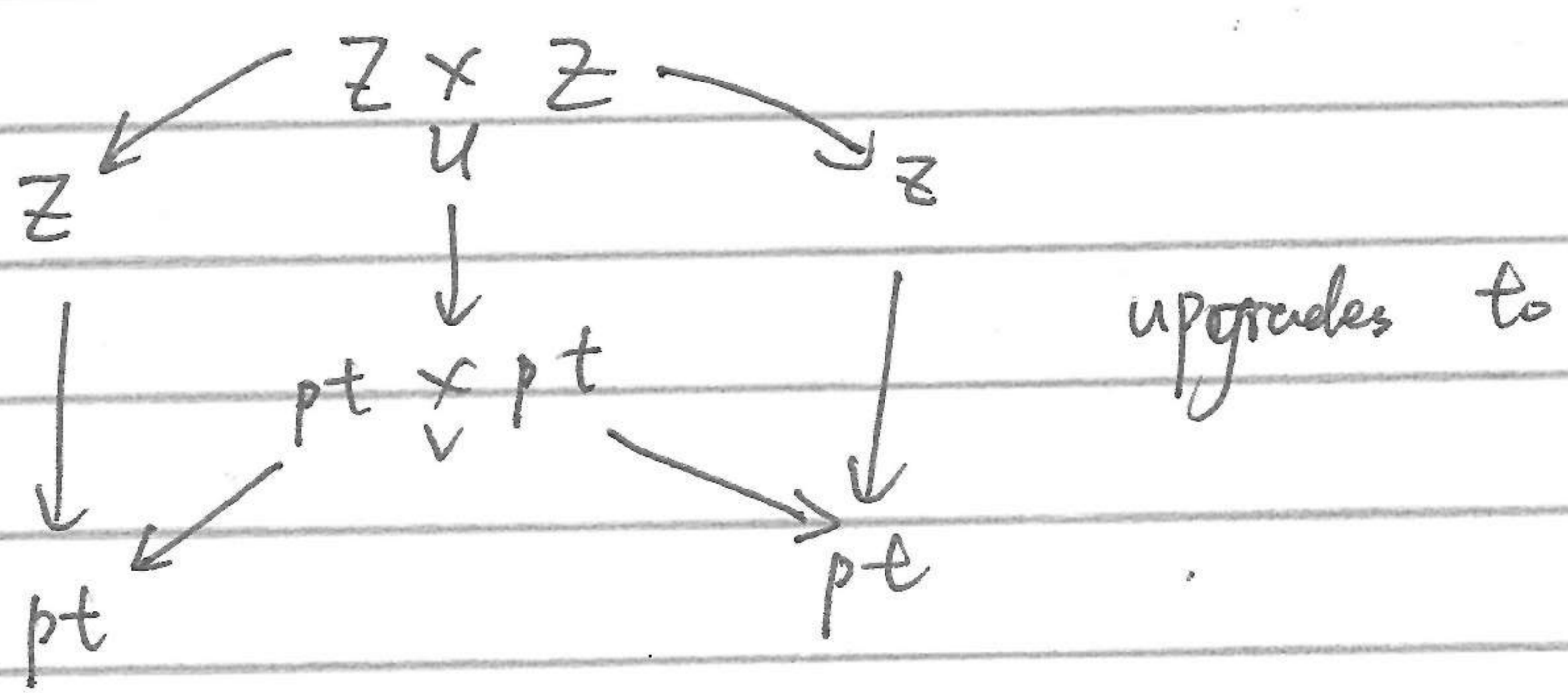
Remark: 1) This is also the SS of the sheaf.

2) $IC(V[-1]) \cong \text{Sym}(V[-2]) \text{-mod}$

right now is not monoidal... need parallelization (because $HC \cong \text{Sym}$ is not yet) (coming up)

Now there's many functorial properties we can prove... or stop. (5.2)

General Picture: Z q-smooth. $Z \xrightarrow{i} U$ U, V smooth affine. $pt \rightarrow V$



(Detail on pg-13)

Observe next that we have maps

$$\begin{array}{ccc}
 \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} & & \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \\
 \text{IC}(p \times p) \times U & & \text{IC}(p \times p) \otimes \mathcal{O}_{U, u} \\
 \downarrow \wr & & \downarrow \wr \\
 \text{IC}(p \times p) \otimes \text{IC}(U) & & \\
 \downarrow \text{id} \otimes \iota^! & & \\
 \text{IC}(p \times p) \otimes \text{IC}(z) & & \\
 \downarrow \wr & & \\
 \text{IC}(z \times_U z) & &
 \end{array}$$

hence E_z -map of the center

$$\text{HC}(p \times p) \otimes T(U, \mathcal{O}_u) \rightarrow \text{HC}(z \times_U z) \rightarrow \text{HC}(z)$$

which will provide us another notion of support of $F \in \text{IC}(z)$ in $\oplus H^{2n}(\text{HC}(p \times p) \otimes T(U, \mathcal{O}_u)) = \text{Sym}(V) \otimes T(U, \mathcal{O}_u)$.

~~(The choice of using U is arbitrary we could say it as~~

$$\text{act}: (p \times p) \times z \rightarrow z$$

or rather its Spec, namely $V^* \times U$.

$$\text{IC}(z) \xrightarrow{\text{act}^!} \text{IC}(p \times p) \otimes \text{IC}(z)$$

~~then $\text{act}^! F$ has~~

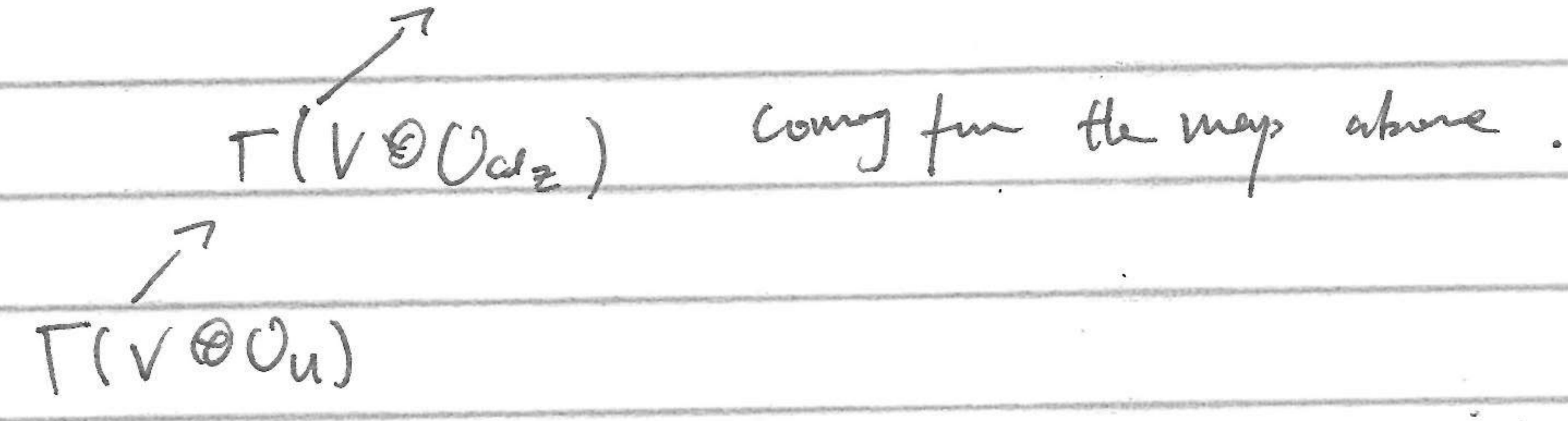
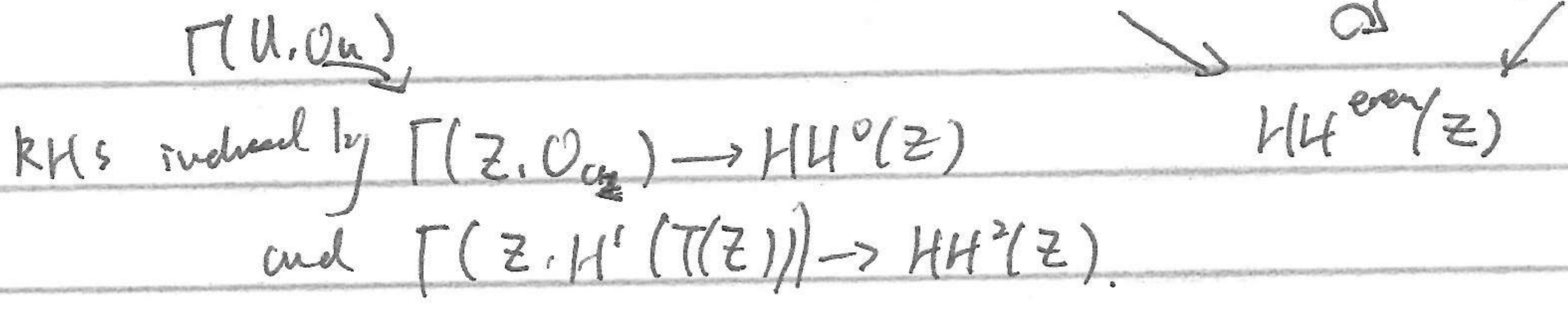
Main Claim:

This support is $\text{SingSupp}(F) \subset \text{Sing}(z) \hookrightarrow V^* \times z \hookrightarrow V^* \times U$.

$$\begin{aligned}
 \text{Reminder: Sing} &= \text{cl} \left(\text{Spec}_z (\text{Sym}_{\mathcal{O}_z} (T(z) \otimes \mathbb{1})) \right) \\
 &= \text{Spec}_{\mathcal{O}_z} (\text{Sym}_{\mathcal{O}_z} (H^1(T(z)|)))
 \end{aligned}$$

and $\text{Sing}(z) \hookrightarrow V^* \times z$ from $T(z) \rightarrow \iota^* T(u) \rightarrow V \otimes \mathcal{O}_z$.
 which in turn comes from pulling res^* $(u \rightarrow v)$ along $z \rightarrow u$.

Namely, one needs to check that $\text{Sym}(V) \otimes \Gamma(U, \mathcal{O}_U) \rightarrow \Gamma(\text{sing } Z, \mathcal{O})$



Now recall if A is E_2 -alg, $A \simeq \text{DGCat } C$, then $Y \subset \text{Spec}(\bigoplus H^m A)$ is such that complement is qc, then $(C_Y \simeq A\text{-mod}_Y \otimes_{A\text{-mod}} C)$. Noetherian condition to make sure $T_Y \rightarrow A$ has continuous right adjoint.

So in particular $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \simeq \mathbb{C} \oplus \mathbb{C}$.
 $IC_Y(Z) \simeq IC(Z) \otimes_{HC(Y/Z)\text{-mod} \otimes QC(U)} (HC\text{-mod} \otimes QC(U))_Y$.

Parallelization

~~Recall~~ Recall $\mathcal{V}[1] = \text{Spec } \text{Sym } V^*[1]$ noncanonically.
 Now fix one such iso (a parallelization).
 Claim: this upgrades $HC(\mathcal{V}[1]) \simeq \text{Sym } V[-2]$ as commutative algebras!
 Corollary: $HC\text{-mod} \simeq \text{Sym } V[-2]\text{-mod}$ as symmetric monoidal cat.

(see: shift in gauge page 12)

We proceed. ~~As $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \simeq \mathbb{C} \oplus \mathbb{C}$~~

The isomorphism is $\text{Sym}(V[-2])^{\text{shift}} = \mathbb{C} \otimes \text{Sym } V$.

Then $S_A = V^*/G_m$.

$$SG_1 \Rightarrow IC(\mathcal{V}[1])_Y = IC(\mathcal{V}[1]) \otimes_{\mathbb{C}(S_A)} \mathbb{C}(S_A)_{Y/G_m}$$

similarly, for the z -relative case:

$$\text{instanton} \mapsto \text{Sym}(V[-2]) \otimes T(U, \mathcal{O}_U)$$

$$\text{shifted: } \text{Sym}(V) \otimes T(U, \mathcal{O}_U)$$

$$S_A = V^*/G_m \times U$$

$$S_G \mid \Rightarrow IC_Y(z) = IC(z) \otimes_{\mathbb{Q}\text{Coh}(S_A)} \mathbb{Q}\text{Coh}(S_A)_Y / G_m \text{ for } Y \subset \text{sing}$$

Application

1) Generators of $IC_Y(z)$.

There's a proper action map

$$\text{act}: Y[-1] \times z \rightarrow z \text{ mentioned before.}$$

$$F := \text{act}^* \overset{IC}{*} : IC(Y[-1] \times z) \xrightarrow{\cong} IC : \text{act}^! =: G, \\ = IC(Y[-1]) \otimes IC(z)$$

G admits a retraction (by pullback along unit) thus conservative thus $\text{im}(F)$ generates.

Another description of F and G :

$$IC(Y[-1]) \otimes IC(z) \cong HC(Y[-1])^{\text{op-mod}} \otimes IC(z) \\ \cong HC(Y[-1])^{\text{op-mod}}(IC(z))$$

One head: $F \in IC(z)$ is a $HC(z)^{\text{op-mod}}$

$$HC(Y[-1]) \rightarrow HC(z/u) \rightarrow HC(z) \text{ so}$$

get $HC(Y[-1])^{\text{op-mod}}$ structure. \square

Other head: $Y[-1] \curvearrowright z$

$$\text{thus } HC(Y[-1])^{\text{op-mod}} \otimes IC(z) \xrightarrow{\text{act}^!} IC(z)$$

\cong

\square

Now we interact w/ \mathcal{G} . We specify what it means to have $\gamma \subset V^* \times U$ support for $F \in IC(\mathcal{V}[-1]) \otimes IC(Z)$.

by giving a map of E_2 -alg:

$$HC(\mathcal{V}[-1]) \otimes T(U, \mathcal{O}_U) \rightarrow HC(IC(\mathcal{V}[-1]) \times Z)$$

This comes from

$$IC(\mathcal{V}[-1]) \otimes \underbrace{QCoh(U)}_{\text{act}} = \underbrace{(HC \otimes T)}_{\text{as } E_2\text{-alg}} \text{-mod}$$

$$IC(\mathcal{V}[-1]) \otimes IC(Z)$$

~~This gives~~

F is compatible w/ $IC(\mathcal{V}[-1]) \otimes QCoh(U)$ -action on everything
 G is lax-compatible but actually compatible.

(key: the action Cat is rigid.)

Then it follows formally that

$$F: (IC(\mathcal{V}[-1]) \otimes IC(Z))_{\gamma} \xrightarrow{\sim} IC_{\gamma \cap \text{sing } Z}(Z)$$

compatible with the $\mathbb{F}_Z^{\gamma, \text{all}}$ functors.

$\gamma \subset \text{sing}(Z)$ then

$$\text{Now } \forall F \in IC_{\gamma}(Z) \Leftrightarrow \mathbb{F}_Z^{\gamma, \text{all}}(F) \xrightarrow{\sim} F$$

$$\Leftrightarrow G(\text{---}) \xrightarrow{\sim} G(F) \text{ conservative}$$

$$\Leftrightarrow G(F) \in IC(\mathcal{V}[-1] \times Z)_{\gamma}$$

and F (resolved) is left adj to G conservative, so

$$F(IC(\mathcal{V}[-1] \times Z)_{\gamma}) \text{ generates } IC_{\gamma \cap \text{sing}(Z)}(Z)$$

$$\text{In particular } IC_{\gamma}(Z) \otimes_{\text{Site}} \gamma \subset \text{sing}(Z)$$

Application in real life: Z aff, g -smooth.
 then $\Xi_Z : \mathcal{O}(Z) \rightarrow IC(Z)$ image is $IC_{\{0\}}(Z)$.

Local, so assume $Z \rightarrow U$
 $\downarrow \quad \downarrow$
 $pt \rightarrow v$

Suffices to check $F_{0\alpha}(IC(\mathcal{Y}[t-1] \times Z)_{\{0\}} \times U)$ has image contained in $\text{im}(\Xi)$.

Further: $p_Z : \mathcal{Y}[t-1] \rightarrow pt$. (claim: $(p_Z \times id)^! : IC(Z) \rightarrow IC(\mathcal{Y}[t-1] \times Z)$ lands in $(\text{---})_{\{0\}} \times U$ and generates.

This is because (generally)

$$IC(\mathcal{Y}[t-1] \times Z)_{0 \times U} = IC(\mathcal{Y}[t-1])_0 \times IC(Z)$$

so suffices to check

$p_Z^! : IC(pt) \rightarrow IC(\mathcal{Y}[t-1])$ lands in IC_0 and generates. This can be done by hand or follows functorially.

so checking image of $IC(Z) \xrightarrow{(p_Z \times id)^!} IC(Z)$

claim: this functor is $L^! \circ \epsilon_*$, $L : Z \rightarrow U$

$Z \times_U Z \rightarrow Z$ base change. so check image of $L^!$

$$\begin{array}{ccc} \downarrow & \downarrow \\ Z & \rightarrow & U \end{array}$$

well, U smooth, so $\mathcal{O}(U) \otimes \omega_U$ generates $IC(U)$, so check $L^!(\omega_U) \in \text{im}(\Xi)$.

$$= \omega_Z$$

\Downarrow
 Gorenstein.

(Content on back side)

shifting game.

Let $C \subset G_m$. $\text{Inv}: G_m\text{-mod} \xrightarrow{\text{(inverses)}} \text{Rep}(G_m)\text{-mod} = \text{mod}$

Observe $\text{Rep}(G_m)$ admits an auto:

$$M \mapsto M^{\text{shift}}, \quad M_{(i,k)}^{\text{shift}} = M_{i+2k,k} \quad (\text{Coh}, G_m)$$

induces $\text{Rep}(G_m)\text{-mod} \xrightarrow{\sim} \text{Rep}(G_m)\text{-mod}$

$C \mapsto C^{\text{shift}}$. (commutes w/ forgetful)

Example: $A: \mathbb{Z}$ -graded. $(A\text{-mod})^{G_m} \xrightarrow{\text{shift}} (A^{\text{shift}}\text{-mod})^{G_m}$

$$(A\text{-mod})^{G_m} \xrightarrow{\text{as plain } D_G} M^{\text{shift}}$$

Warning: for $G_m\text{-mod}$, this shifting is not oblv to $D_G \text{Cat}: (A\text{-mod})^{\text{shift}} = A^{\text{shift}}\text{-mod}$

Suppose $A \in \text{Ex-alg}(\text{Rep}(G_m))$

and A^{shift} is classical then it is Ex thur so is A .

(shift is sym. monoidal)

By above, we have

$$(A\text{-mod})^{G_m} = (A^{\text{shift}}\text{-mod})^{G_m} = \mathcal{Q}\text{Coh}(\text{Spec}(A^{\text{shift}}/G_m))$$

Next, observe A^{shift} classical $\Rightarrow A^{\text{shift}} \simeq \bigoplus H^{2n}(A)$ as classical.

Then $Y \subset \text{Spec}(\bigoplus H^{2n} A)$ conical

$\Rightarrow Y/G_m \hookrightarrow S_A$ makes sense.

6.1 Prop.

$$A\text{-mod}_Y = A\text{-mod} \otimes_{\mathcal{Q}C(S_A)} \mathcal{Q}C(S_A)_{Y/G_m}$$

$$\text{where } A\text{-mod} = A\text{-mod} \otimes_{(A\text{-mod})^{G_m}} A\text{-mod}^{G_m} \simeq A\text{-mod} \otimes_{\mathcal{Q}C(S_A)} \mathcal{Q}C(S_A)$$

~~Supp~~ (I'll prove later) - back page

Suppose now $A\text{-mod} \rightarrow C$ some cat. Then

$\mathcal{Q}\text{Coh}(S_A) \rightarrow C$ as well. Let $Y \subset \text{Spec}(A)$, conical, complement of

then as showed before, $C_Y \simeq C \otimes_{\mathcal{Q}C(S_A)} \mathcal{Q}(S_A)_{Y/G_m}$.

and no further (ok)

Proof: ~~A mod~~ By de-conv. argument suffices to check

$$(A\text{-mod})_{G_m} \text{ matches up } Q(SA) \gamma / G_m$$

$$\text{under } (A\text{-mod})_{G_m} \cong Q(SA)$$

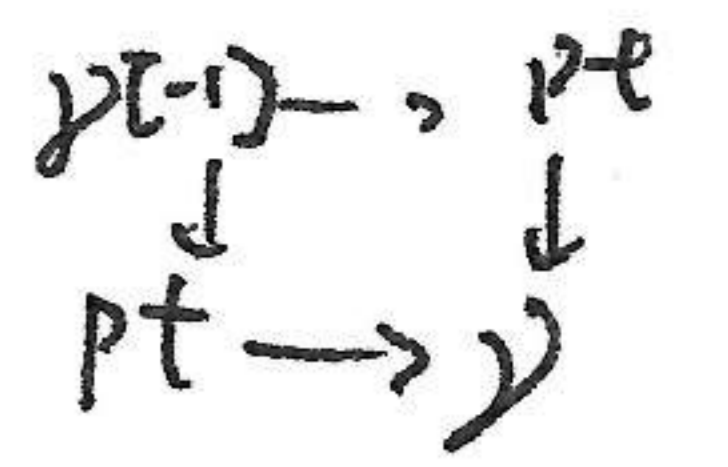
(Note: need to check $A\text{-mod}_{G_m}$ makes sense.

γ convex ~~is not~~ is not convex grading.

$A\text{-mod}_{G_m}$ is invariant $\Leftrightarrow \gamma$ convex not extra grading.

But A shift classical means it's same grading!

So what's $(A\text{-mod})_{G_m}$? It's graded A -modules whose support is on γ . ~~The rest~~ The rest is formal.



pg 3: $\mathcal{Y}[-1]$

\mathcal{Y} affine & smooth.
~~classical.~~

Observation: A. it's quasi-smooth. (by definition)
 B. it is ~~not~~ canonically isomorphic to $\text{Spec Sym } V^*[c]$, where $V = \mathcal{K}_{T_{pt} \mathcal{Y}}$. (Worry not, \mathcal{Y} is classical!)

B: suffices to restrict to formal neighborhood.

Non-canonically, this is A^n (exponential of V).

This now boils down to a pushout in ComAlg , for which it's straightforward.

pg. 6

~~Prop: $\mathcal{Y} \subseteq \text{Spec } A$, $\text{Spec } A \setminus \mathcal{Y}$ is $\mathcal{Y} \rightarrow T_{\mathcal{Y}}$ compactly generated.~~

~~Locally, $\text{Cone}(t_0 \rightarrow t)$ generate.~~

~~Armed support in means support of \mathcal{Y} as \mathcal{Y} is \mathcal{Y} .~~

Prop: \mathcal{Y} arbitrary $\Rightarrow \forall t$. $\text{supp } t \subset \mathcal{Y} \iff \text{supp Hom}_T^*(t_0, t) \subset \mathcal{Y} \forall a$.
 $\{t_a\}$ generators. compact. as A -mod.

(Recall: $A \rightarrow \bigoplus_{u \geq 0} \text{Hom}(t, t[2u])$ so it acts on $\text{Hom}^*(t_0, t)$ (derived cat) by composition.

Pf:

Let a be $\in A$, homogeneous of deg $2k$.

$$\text{supp}(t) \subset \mathcal{Y}_a \iff \text{colim}(t \xrightarrow{a} t[2k] \xrightarrow{a} \dots) = 0.$$

$$\iff \text{Hom}(t_\alpha[m], \text{colim}) = 0 \quad \forall \alpha, m.$$

$$\iff \text{colim}(\text{Hom}(t_\alpha[m], -)) = 0 \quad \forall \alpha, m.$$

$$\iff \text{supp Hom}_T^*(t_\alpha, t) \subset \mathcal{Y}_a \quad \forall \alpha.$$

Now observe $\mathbb{I}(\mathcal{P}^t, \mathcal{Y}^t)$ has one compact gen: $\Delta_{\mathcal{Y}}^{\mathbb{I}^c}(K)$.
 Then claim is trivial.